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GENDER-DIFFERENTIATED HUMAN CAPITAL AND TIME DISTRIBUTIONS IN A GENERALIZED HECKSCHER-OHLIN MODEL WITH ENDOGENOUS PHYSICAL CAPITAL

Wei-Bin ZHANG

Ritsumeikan Asia Pacific University, 1-1 Jumonjibaru, Beppu-Shi, Oita-ken, 74-8577 Japan, e-mail: wbz1@apu.ac.jp

Abstract *This paper deals with the role of preferences and technological differences between any number of national economies in determining the dynamics of capital stocks, gender-differentiated human capital, pattern of trade and gender-differentiated time distribution in a reformed H-O model. The trade model studies endogenous wealth and human capital accumulation and labor and capital distribution between sectors and between countries under perfectly competitive markets and free trade. It is an integration of the H-O model, the Solow-Uzawa neoclassical growth model, the Uzawa-Lucas two-sector model, and the Oniki-Uzawa trade model. It synthesizes these well-known economic models with Zhang's utility function for describing household behavior. The paper simulates the model, demonstrating the motion of the dynamic system and the existence of equilibrium point. We also examine the effects of changes in the total factor productivity of a country's industrial sector, the productivity of country's education sector, one country's propensity to consume another country's global commodity, man's propensity to stay at home, woman's propensity to receive education, and a country 1's population.*

Key words:

trade pattern; O-H model; Oniki-Uzawa model; Uzawa-Lucas model; gender; time distribution; learning by education; propensity to receive education

1. Introduction

Contempory conomic transformations have been characterized of increasing proportion of hight education among female population and entry of women into the labor force. Nevertheless, there are only a few formal economic models which explicitly deal with interdependence between gender-based human capital, time distribution and economic growth. The purpose of this study is to address issues related to dynamic interactions between global economic growth, wealth accumulation, gender-differenciated education and human capital in a with endogenous physical and human capital accumulation in a reformed Heckscher-Ohlin model (Heckscher, 1919; Ohlin, 1933; and Heckscher and Ohlin, 1991). It is an integration of the H-O model, the Solow-Uzawa neoclassical growth model (Solow, 1956; and Uzawa, 1961, 1963), the Uzawa-Lucas two-sector model (Uzawa, 1965, Lucas, 1988), and the Oniki-Uzawa trade model (Oniki and Uzawa, 1965). It synthesizes these well-known economic models with Zhang's utility function for describing household behavior (Zhang, 1993).

In order to explain economic growth a formal economic theory should take account of economic structual changes. As emphasized by D'Agata (2009: 1), "any *theoretically* adequate model of structural dynamics has to explain the growth (or decay) of sectors and their technological dynamics in terms of process and production innovations as outcome of rational agents, and the changes of consumption vectors as the result of changes of consumer's knowledge." The traditional neoclassical growth theory has not adequately modeled economic structural change on the basis of microeconomic foundation (Jones and Manuelli, 1997; Buirmesiter and Dobell, 1970; Zhang, 2005). It is well known that most of the models in the neoclassical growth theory model are extensions and generalizations of the pioneering works of Solow in 1956. The model has played an important role in the development of economic growth theory by using the neoclassical production function and neoclassical production theory. Solow's one-sector model has been extended to the case of two-sector economies by, for instance, Uzawa (1961, 1963). The Uzawa model considers that output of the capital sector goes entirely to investment and that of the consumption sector entirely to consumption. This assumption avoids the problem of modeling consumers' choice among goods and services in growth theory. There is a single commodity for consumer in the Uzawa model. The Solow-Uzawa model has been extended and generalized in different directions (e.g., Diamond, 1965; Stiglitz, 1967; Gram, 1976; Mino, 1996; Drugeon and Venditti, 2001; Ladrón-de-Guevara *et.al*. 1999; Farmer and Wendner, 2003). We model the national economic structure on the basis of the Solow-Uzawa model.

In modelling international trade, we integrate the Heckscher-Ohlin (H-O) model and the Oniki-Uzawa model. The H-O model is one of the core models in model trade theories. A standard H-O model is for a two-country global economy, each country having access to the same technology for producing two goods using two fixed factors (labor and capital) under conditions of perfect competition and constant returns to scale. Factors of production are mobile between sectors within a country, but immobile internationally. No international borrowing and lending are allowed. The framework is obviously invalid for modern economies as capital is mobile between countries. The H-O model explains patterns of trade based on the factor endowments of countries. According to Chen (1992: 923-4), "It appears to be the general consensus in this body of literature that the main determinant of long-run comparative advantage is the countries' savings rates. The question of what has caused the difference in savings rates among countries, however, is rarely explicitly discussed in the literature." This study also considers endogenous wealth as a engine of global economic growth. The wealth accumulating is influenced by the Oniki-Uzawa (Oniki and Uzawa, 1965; Frenkel and Razin, 1987; Sorger, 2002; and Nishimura and Shimomura, 2002). The Oniki-Uzawa model is constructed for the two-country with two goods with fixing saving rates. Deardorff and Hanson (1978) construct a model of different fixed rates in which the country with the higher savings rate exports the capital intensive good in steady state. In most of this type models goods and services are classified into capital goods and consumer goods. Nevertheless, it has been recorded that a high share of GDP in modern economies is non-tradable (Mendoza, 1995). Backus and Smith (1993:1) emphasized the significance of this distinction as follows: "The mechanism is fairly simple. "Although the law of one price holds, in the sense that each good sells for a single price in all countries, PPP may not: price indexes combine prices of both traded and nontraded goods, and because the latter are sold in only one country their prices, and hence price indexes, may differ across countries." There are also other studies emphasizing distinction between tradable and non-tradable sectors (Stockman and Tesar, 1995; Zhao et al., 2014). This study introduces distinct sectors to examine trade patterns and economic dynamics.

We analyze female education and the entry of women into the labor force as endogenous processes of global economic growth. Bar and Leukhina (2011) observe that married females more than doubled their workforce participation in the last half a century. There are many empirical studies on issues related to labor market and economic development with gender (Blau and Kahn, 2000; Stotsky 2006; Croson and Gneezy, 2009). According to Eckstein and Lifshitz (2011), the rise in education levels accounts for about 33 percent of the increase in female employment, and the rise in wages and narrowing of the gender wage gap account for another 20 percent, while about the 40 percent remains unexplained by observed household characteristics. Although there are many studies on gender differences in education and growth, as pointed out by Bandiera and Natraj (2013), these studies are of limited use for revealing relationships between education and growth as they often do not identify the causal link from gender differences to economic growth. Human capital accumulation is modeled according to the approach by Uzawa (1965) and Lucas (1988). This study extends the Uzawa-Lucas two-sector model to include gender difference. Dolado *et al*. (2001) compare the incidence and composition of female employment both in the EU and in the US. Female participation rates had noticeably increased in the EU and the US since the 1960s (e.g., Antecol, 2000; Biagetti and Sergio, 2009; Fogli and Veldkamp, 2011). Some formal economic theories are proposed to explain why the changes in female labor participation take place and examine the factors which are significant determinants of the dynamics (e.g., Becker, 1985; Chiappori, 1992; Gomme *et al*. 2001; Campbell and Ludvigson, 2001; Gutierrez, 2003; Tassel, 2004; Fernández, 2007; Trede and Heimann, 2011). In particular, implications of economic rationality in the allocation of time and gender issues have been made explicit in a formal and rigorous theory of the subject since Becker (1965) published his seminal work in 1965. There is an immense body of theoretical literature on economic growth with time distribution between home and non-home economic and leisure activities (e.g., Greenwood and Hercowitz, 1991; Benhabib and Perli, 1994; Turnovsky, 1999; Rupert *et al*. 2001; and Vendrik, 2003). Inspired by the richness of empirical studies and influenced by different formal economic models, this study develops an integrated analytical framework to study endogenous labor supply and gender division of labor with endogenous human and physical capital accumulation in a multi-country framework. It should be remarked that the model in this study is an integration of the two models by Zhang. Zhang (2014) proposes a growth model with gender-differentiated human capital and family wealth accumulation based on the Uzawa-Lucas two-sector model for a national economy. Zhang (2015) makes a synthesis of the Heckscher-Ohlin and Oniki-Uzawa trade models with heterogeneous tastes, different technologies, and endogenous wealth for a two-country economy. This paper develops an international growth model with three sectors in each national economy for a global economy with any number of national economies. The rest of the paper is organized as follows. Section 2 defines the basic model. Section 3 shows how we solve the dynamics and simulates the motion of the global economy. Section 4 carries out comparative dynamic analysis to examine the impact of changes in some parameters on the motion of the global economy. Section 5 concludes the study. The appendix proves the main results in Section 3.

2. The Model

The model in this study is influenced by some typical dynamic H-O models and the neoclassical trade growth theory and is based on Zhang (2014, 2015). We consider a world economy with any number of national economies which are indexed by $j = 1, ..., J$. Different from the standard H-O model which deals with a world economy with only two countries and two goods, this study assumes that each national economy produces three goods. Each country produces a homogeneous capital consumer good which can be used as capital and consumption. This sector is called industrial sector. The industrial sector is similar to the homogenous sector in the traditional neoclassical trade model (e.g., Ikeda, and Ono, 1992). Capital goods are freely mobile between countries. There is no tariff on any good in the global economy. Each country produces services which are not internationally tradable and are consumed only by domestic households. Each country specifies in producing a good called global commodity which is internationally tradable. Global commodities are pure consumption goods. Production sectors use capital and labor as inputs. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Like in the traditional H-O model, labor is internationally immobile. Capital and labor are freely mobile within each country and are immobile between countries. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production. We omit the possibility of hoarding of output in the form of non-productive inventories held by households.

Following the Uzawa-Lucas model, we introduce education sectors to the economic system. Let all the prices be measured in terms of capital good. We use subscripts, i, s, m, and e to denote industrial sector, service sector, country j's global commodity sector, and education sector, respectively. We use (j, q) to index sector q, $q = i, s, m, e$, in country j. Let $p_{jq}(t)$ stand for the price of product (j, q). As labor is immobile between countries, wages vary between countries. Markets are competitive; thus labor and capital earn their marginal products, and firms earn zero profits. We denote the wage rate in the j th country and rate of interest by $w_j(t)$ and $r(t)$, respectively. Capital depreciation rates vary between countries. Capital depreciates at a constant exponential rate δ_{jk}. We use $N_{jq}(t)$ and $K_{jq}(t)$ to stand for the labor force and capital stocks employed by sector q in country j. Let $F_{jq}(t)$ stand for the output level of sector q in country j.

The population of each gender is homogeneous in any country. We assume that each family consists of husband and wife. As all the families are identical, the family structure is invariant over time under these assumptions. We make the population structure with the same reason as given by Albanesi and Olivetti (2009: 82): "Since the purpose of this paper is to study the joint determination of gender differentials in labor market outcomes and in the household division of labor, we abstract from modelling marriage decisions ...". The population is constant. We use $\bar{N}_j$ to stand for the number of men. We use subscripts $g = 1$ and $g = 2$ to stand for man and woman respectively. Let $T_{j1}(t)$ and $T_{j2}(t)$ stand for the work time of husband and wife of a representative household in country j and $N_j(t)$ for the flow of labor services used at time t for production. In this study we assume that gender differences in productivity and social activities are measured by differences in human capital and preference. We have $N_j(t)$ as follows

$$N_j(t) = \left[H_{j1}^{\theta_{j1}}(t) T_{j1}(t) + H_{j2}^{\theta_{j2}}(t) T_{j2}(t) \right] \bar{N}_j , \qquad (1)$$

where $H_{jg}(t)$ is the level of human capital of gender g and θ_{jg} is gender g's human capital utilization efficiency parameter. We call $H_{jg}^{\theta_{jg}}(t)$ gender g's level of effective human capital. The labor force is distributed between the four sectors.

The production functions
The production functions are neoclassical and homogeneous of degree one with labor and physical capital as inputs. The production function of section q in country j is specified as

$$F_{jq}(t) = A_{jq} K_{jq}^{\alpha_{jq}}(t) N_{jq}^{\beta_{jq}}(t), \; A_{jq}, \alpha_{jq}, \beta_{jq} > 0, \; \alpha_{jq} + \beta_{jq} = 1, \tag{2}$$

where A_{jq}, α_{jq}, and β_{jq} are positive parameters. It should be noted that the traditional H-O model assumes variations in capital and labor endowments with the identical technology between countries. In this study we assume not only differences in capital and labor endowments like in the H-O model but also differences in technologies between countries.

The marginal conditions
The rate of interest, wage rate, and prices are determined by markets. Hence, for any individual firm rate of interest, wage rate, and prices are given at each point of time. The industrial sector chooses the two variables $K_{ji}(t)$ and $N_{ji}(t)$ to maximize its following profit

$$p_{jq}(t) F_{jq}(t) - (r(t) + \delta_{jk}) K_{jq}(t) - w_j(t) N_{jq}(t).$$

where $p_{ji}(t) = 1$. The marginal conditions are

$$r(t) + \delta_{jk} = \frac{\alpha_{jq} p_{jq}(t) F_{jq}(t)}{K_{jq}(t)}, \; w_j(t) = \frac{\beta_{jq} p_{jq}(t) F_{jq}(t)}{N_{jq}(t)}. \tag{3}$$

According to the definitions, we have

$$w_{jg}(t) = H_{jq}^{\theta_{jq}}(t) w_j(t). \tag{4}$$

Behavior of consumers
This study uses an alternative approach to the Ramsey approach to consumer behaviour proposed by Zhang (1993, 2005). The households choose consumption levels of services and commodities, education time and leisure time, and amount of saving. We use $\bar{k}_j(t)$ to stand for wealth per household, i.e., $\bar{k}_j(t) = \bar{K}_j(t) / \bar{N}_j$, where $\bar{K}_j(t)$ is $j's$ total physical wealth. To explain our approach to consumer behaviour, first define per capita current income $y_j(t)$ from the interest and wage payments as follows

$$y_j(t) = r(t) \bar{k}_j(t) + w_{j1}(t) T_{j1}(t) + w_{j2}(t) T_{j2}(t).$$

The "disposble" income is not necessarily equal to the current income because consumers can sell wealth to pay, for instance, current consumption if the current income is not sufficient for purchasing goods and services. The total value of wealth that a representative household can sell to purchase goods and to save is equal to $\bar{k}_j(t)$. We assume that selling and buying wealth can be conducted instantaneously without any transaction cost. This is evidently a strict consumption as it may take time to draw savings from bank or to sell one's properties. The per capita disposable income of the household is defined as the sum of the current income and the wealth available for purchasing consumption goods and saving

$$\hat{y}_j(t) = y_j(t) + \bar{k}_j(t).$$

At each point in time, each household distributes the total available budget between saving, $s_j(t)$, consuming capital good $c_{ji}(t)$, consuming global commodities $c_{jm}(t)$, a consuming capital good $c_{js}(t)$, and receiving education, $\tilde{T}_{j1}(t)$, and $\tilde{T}_{j2}(t)$. The budget constraint is given by

$$c_{ji}(t) + p_{js}(t)c_{js}(t) + s_j(t) + p_{je}(t)\tilde{T}_{j1}(t) + p_{je}(t)\tilde{T}_{j2}(t) + \sum_{q=1}^{J} p_{qm}(t)c_{jq}(t) = \hat{y}_j(t). \quad (5)$$

This equation means that consumption and savings exhaust the consumers' disposable income. Let $\bar{T}_{jg}(t)$ stand for the leisure time of gender g at time t. A person of gender g in any country is faced with the time constraint

$$T_{jg}(t) + \tilde{T}_{jg}(t) + \bar{T}_{jg}(t) = T_0,$$

where T_0 is the total available time for work, education and leisure. Substituting this function into (5) yields

$$\begin{aligned} & c_{ji}(t) + p_{js}(t)c_{js}(t) + s_j(t) + \bar{p}_{j1}(t)\tilde{T}_{j1}(t) + \bar{p}_{j2}(t)\tilde{T}_{j2}(t) + w_{j1}(t)\bar{T}_{j1}(t) + w_{j2}(t)\bar{T}_{j2}(t) \\ & + \sum_{j=}^{J} p_{qm}(t)c_{jq}(t) = \bar{y}_j(t), \end{aligned} \quad (6)$$

where we use $\hat{y}_j(t) \equiv y_j(t) + \bar{k}_j(t)$, and

$$\bar{p}_{jg}(t) \equiv p_j(t) + w_{jg}(t),$$
$$\bar{y}_j(t) \equiv (1 + r(t))\bar{k}_j(t) + w_{j1}(t)T_0 + w_{j2}(t)T_0.$$

The left-hand side of (5) is the sum of the cost of consumption, saving, and opportunity costs of leisure and education times. At each point in time, the household decides the $7 + J$ variables. We assume that the utility level $U_j(t)$ is dependent on the education times, the leisure times, the consumption level of commodity, and the saving, as follows

$$U_j(t) = u_j(t)\bar{T}_{j1}^{\sigma_{j01}}(t)\bar{T}_{j2}^{\sigma_{j02}}(t)\tilde{T}_{j1}^{\eta_{j01}}(t)\tilde{T}_{j2}^{\eta_{j02}}(t)c_{ji}^{\xi_{j0}}(t)c_{js}^{\chi_{j0}}(t)s_j^{\lambda_{j0}}(t)\prod_{q=1}^{J} c_{jq}^{\gamma_{j0q}}(t),$$
$$\sigma_{j0g}, \eta_{j0g}, \xi_{j0}, \chi_{j0}, \lambda_{j0}, \gamma_{j0q} \geq 0.$$

where u_j, is a time-dependent variable, σ_{j0g}, η_{j0g} are called, respectively gender $g's$ propensities to use leisure time and to receive education, and ξ_{j0}, χ_{j0} and γ_{j0q}, respectively, the household's propensities to consume good, service, and country $q's$ global commodity, and λ_{j0} the household's propensity to hold wealth. Maximizing U_j subject to budget constraint (4) yields

$$\begin{aligned} & w_{jg}(t)\bar{T}_{jg}(t) = \sigma_{jg}\bar{y}_j(t), \ \bar{p}_{jg}(t)\tilde{T}_{jg}(t) = \eta_{jg}\bar{y}_j(t), \ c_{ji}(t) = \xi_j\bar{y}_j(t), \ p_{js}(t)c_{js}(t) = \chi_j\bar{y}_j(t), \\ & p_{qm}(t)c_{jq}(t) = \gamma_{jq}\bar{y}_j(t), \ s_j(t) = \lambda_j\bar{y}_j(t), \end{aligned} \quad (7)$$

where

$$\sigma_{jg} \equiv \sigma_{jg}\rho_j, \ \eta_{jg} \equiv \eta_{j0g}\rho_j, \ \xi_j \equiv \xi_{j0}\rho_j, \ \chi_j \equiv \chi_{j0}\rho_j, \ \lambda_j \equiv \lambda_{j0}\rho_j, \ \gamma_{jq} \equiv \gamma_{j0q}\rho_j,$$

$$\rho_j \equiv \frac{1}{\sigma_{j01} + \sigma_{j02} + \eta_{j01} + \eta_{j02} + \xi_{j0} + \chi_{j0} + \lambda_{j0} + \sum_{q=1}^{J}\gamma_{j0q}}.$$

Accumulation of human capital

In this study, we follow the Uzawa-Lucas model in modeling human capital accumulation. We assume that human capital accumulation is through education. We propose the following human capital accumulation equation (Zhang, 2007, 2014)

$$\dot{H}_{jg}(t) = \frac{\upsilon_{jge}\left(F_{je}(t)/2\bar{N}_j(t)\right)^{a_{jge}}\left(H_{jg}^{\theta_{jg}}(t)\tilde{T}_{jg}(t)\right)^{b_{jge}}}{H_{jg}^{\pi_{jge}}(t)} - \delta_{jgh}H_{jg}(t), \ g = 1, 2, \tag{8}$$

where $\delta_{jgh}\ (>0)$ is the depreciation rate of human capital, υ_{jge}, a_{jge}, and b_{jge} are non-negative parameters. The equations imply that human capital tends to increase in the level of education, $F_{je}/2\bar{N}_j$, and in the (qualified) study time, $H_{jg}^{\theta_{jg}}\tilde{T}_{jg}$. The term $H^{\pi_{jge}}$ indicates that as the level of human capital increases, it may be more difficult (in the case of π_{qe} being large) or easier (in the case of π_{qe} being small) to accumulate more human capital via formal education. It should be noted that in the literature on education and economic growth, it is assumed that human capital evolves according to the following equation (Lucas, 1988)

$$\dot{H}(t) = H^{\eta}(t)\, G(T_e(t))$$

where the function G is increasing as the effort rises with $G(0) = 0$. It seems reasonable to consider diminishing returns in human capital accumulation: people accumulate it rapidly early in life, then less rapidly, then not at all – as though each additional percentage increment were harder to gain than the preceding one. Solow (2000) adapts the Uzawa formation to the following form

$$\dot{H}(t) = H(t)\,\kappa T_e(t).$$

This equation implies that if no effort is devoted to human capital accumulation, then $\dot{H}(t) = 0$ (human capital does not vary as time passes); if all effort is devoted to human capital accumulation, then $g_H(t) = \kappa$ (human capital grows at its maximum rate; this results from the assumption of potentially unlimited growth of human capital). Between the two extremes, there is no diminishing return to the stock $H(t)$. To achieve a given percentage increase in $H(t)$ requires the same effort. As remarked by Solow, the above formulation is very far from a plausible relationship. If we consider the above equation as a production for new human capital (i.e., $\dot{H}(t)$), and if the inputs are already accumulated human capital and study time, this production function is homogenous of degree two. It has strong increasing returns to scale and constant returns to $H(t)$ itself. It can be seen that our approach is more general to the traditional formation with regard to education.

Wealth accumulation

According to the definitions of $s_j(t)$, the wealth change of the representative household in country j is

$$\dot{\bar{k}}_j(t) = s_j(t) - \bar{k}_j(t). \tag{9}$$

This equation simply states that the change in wealth is equal to saving minus dissaving.

Factor marketing clearing conditions

We use $K_j(t)$ to stand for the capital stocks employed by country j. The capital stock is fully employed by the four sectors. That is

$$K_{ji}(t) + K_{jm}(t) + K_{js}(t) + K_{je}(t) = K_j(t), \quad j = 1, \ldots, J. \tag{10}$$

The labor force is fully employed by the four sectors

$$N_{ji}(t) + N_{jj}(t) + N_{js}(t) + N_{je}(t) = N_j(t). \tag{11}$$

Market clearing for services
The demand and supply of services balance in each national market

$$c_{js}(t) N_j = F_{js}(t). \tag{12}$$

Market clearing for education
For the education sector, the demand for and supply of education balance at any point in time

$$\tilde{T}_{j1}(t) N_j + \tilde{T}_{j2}(t) N_j = F_{je}(t), \quad j = 1, \ldots, J. \tag{13}$$

Market clearing in global commodity markets
The demand and supply of tradable goods balance in global markets

$$\sum_{j=1}^{J} c_{jm}(t) N_j = F_{jm}(t). \tag{14}$$

Market clearing in capital markets
The global capital production is equal to the global net savings. That is

$$\sum_{j=1}^{J} \left(s_j(t) N_j - \bar{k}_j(t) N_j + c_{ji}(t) N_j + \delta_{kj} K_j(t) \right) = \sum_{j=1}^{J} F_{ji}(t). \tag{15}$$

Wealth balance
The wealth owned by the global population is equal to the total global wealth

$$\sum_{j=1}^{J} \bar{k}_j(t) N_j = \sum_{j=1}^{J} K_j(t) = K(t). \tag{16}$$

We thus built the dynamic model with endogenous wealth accumulation.

3. The Dynamics and Equilibrium

We have the dynamic equations for the global economy. As the system is nonlinear and is of high dimension, it is difficult to generally analyze behavior of the system. Before examining the dynamic properties of the system, we show that dynamics of the global economy can be expressed by $3J$ differential equations. First, we introduce a variable

$$z_1(t) \equiv \frac{r(t) + \delta_{1k}}{w_1(t)}.$$

The following lemma shows how to follow the dynamics of global economic growth with initial conditions.

Lemma
The motion of the $3J$ variables, $z_1(t)$, $\bar{k}_j(t)$ and $H_{jg}(t)$, where $\bar{k}_j(t) = (\bar{k}_2(t), \ldots, \bar{k}_J(t))$, are given by the following $3J$ differential equations

$$\dot{z}_1(t) = \tilde{\Lambda}_1\left(z_1(t), \{\bar{k}_j(t)\}, \{H_j(t)\}\right),$$
$$\dot{\bar{k}}_j(t) = \tilde{\Lambda}_j\left(z_1(t), \{\bar{k}_j(t)\}, \{H_j(t)\}\right), \quad j = 2, ..., J,$$
$$\dot{H}_{jg}(t) = \Lambda_{jg}\left(z_1(t), \{\bar{k}_j(t)\}, \{H_j(t)\}\right) \qquad (17)$$

where $\tilde{\Lambda}_j(t)$ and $\Lambda_{jg}(t)$ are functions of $z_1(t)$, $\{\bar{k}_j(t)\}$, and $\{H_{jg}(t)\}$, defined in the appendix. The values of the other variables are given as functions of $z_1(t)$, $\{\bar{k}_j(t)\}$, and $H_{jg}(t)$ at any point in time by the following procedure: $r(t)$ by (A2) $\rightarrow$ $w_j(t)$ by (A2) $\rightarrow$ $z_j(t)$ by (A3) $\rightarrow$ $p_{jq}(t)$ by (A4) $\rightarrow$ $w_{jg}(t)$ by (A5) $\rightarrow$ $\bar{k}_1(t)$ by (A19) $\rightarrow$ $\bar{y}_j(t)$ by (A5) $\rightarrow$ $K(t)$ by (A17) $\rightarrow$ $K_j(t)$ by (A16) $\rightarrow$ $N_{ji}(t)$ by (A15) $\rightarrow$ $N_{jm}(t)$ by (A14) $\rightarrow$ $N_j(t)$ by (A12) $\rightarrow$ $N_{je}(t)$ by (A8) $\rightarrow$ $N_{js}(t)$ by (A7) $\rightarrow$ $K_{jq}(t)$ by (A1) $\rightarrow$ $F_{jq}(t)$ by (2) $\rightarrow$ $c_{ji}(t)$, $c_{jm}(t)$, $\bar{T}_{jg}(t)$, $\tilde{T}_{jg}(t)$ and $s_j(t)$ by (7).

For simulation, we specify values of the parameters as follows

$$T0 = 1,\ \xi_{j0} = 0.05,\ \gamma_{j0q} = 0.01,\ \gamma_{j0j} = 0.02,\ j \neq q,$$
$$a_{jqe} = 0.3,\ b_{jqe} = 0.5,\ \pi_{jge} = 0.15,\ \delta_{j1h} = 0.03,\ \delta_{j2h} = 0.035,$$
$$\begin{pmatrix} \bar{N}_1 \\ \bar{N}_2 \\ \bar{N}_3 \end{pmatrix} = \begin{pmatrix} 100 \\ 200 \\ 300 \end{pmatrix},\ \begin{pmatrix} A_{1i} \\ A_{2i} \\ A_{13} \end{pmatrix} = \begin{pmatrix} 1.4 \\ 1.3 \\ 1.2 \end{pmatrix},\ \begin{pmatrix} A_{1s} \\ A_{2s} \\ A_{3s} \end{pmatrix} = \begin{pmatrix} 1 \\ 0.9 \\ 0.8 \end{pmatrix},\ \begin{pmatrix} A_{1e} \\ A_{2e} \\ A_{3e} \end{pmatrix} = \begin{pmatrix} 1.4 \\ 1.2 \\ 1 \end{pmatrix},\ \begin{pmatrix} A_{1m} \\ A_{2m} \\ A_{3m} \end{pmatrix} = \begin{pmatrix} 1.3 \\ 1.2 \\ 1.1 \end{pmatrix},\ \begin{pmatrix} \delta_{1k} \\ \delta_{2k} \\ \delta_{3k} \end{pmatrix} = \begin{pmatrix} 0.05 \\ 0.05 \\ 0.06 \end{pmatrix},$$
$$\begin{pmatrix} \alpha_{1i} \\ \alpha_{2i} \\ \alpha_{13} \end{pmatrix} = \begin{pmatrix} \alpha_{1s} \\ \alpha_{2s} \\ \alpha_{3s} \end{pmatrix} = \begin{pmatrix} \alpha_{1e} \\ \alpha_{2e} \\ \alpha_{3e} \end{pmatrix} = \begin{pmatrix} \alpha_{1m} \\ \alpha_{2m} \\ \alpha_{3m} \end{pmatrix} = \begin{pmatrix} 0.32 \\ 0.31 \\ 0.3 \end{pmatrix},\ \begin{pmatrix} \theta_{11} \\ \theta_{12} \\ \theta_{21} \end{pmatrix} = \begin{pmatrix} 0.75 \\ 0.7 \\ 0.7 \end{pmatrix},\ \begin{pmatrix} \theta_{22} \\ \theta_{31} \\ \theta_{32} \end{pmatrix} = \begin{pmatrix} 0.65 \\ 0.65 \\ 0.6 \end{pmatrix},\ \begin{pmatrix} \sigma_{101} \\ \sigma_{201} \\ \sigma_{301} \end{pmatrix} = \begin{pmatrix} 0.14 \\ 0.16 \\ 0.18 \end{pmatrix},$$
$$\begin{pmatrix} \sigma_{102} \\ \sigma_{202} \\ \sigma_{303} \end{pmatrix} = \begin{pmatrix} 0.15 \\ 0.17 \\ 0.2 \end{pmatrix},\ \begin{pmatrix} \eta_{101} \\ \eta_{201} \\ \eta_{301} \end{pmatrix} = \begin{pmatrix} 0.02 \\ 0.016 \\ 0.012 \end{pmatrix},\ \begin{pmatrix} \eta_{102} \\ \eta_{202} \\ \eta_{302} \end{pmatrix} = \begin{pmatrix} 0.015 \\ 0.014 \\ 0.01 \end{pmatrix},\ \begin{pmatrix} \chi_{10} \\ \chi_{20} \\ \chi_{30} \end{pmatrix} = \begin{pmatrix} 0.07 \\ 0.06 \\ 0.05 \end{pmatrix},\ \begin{pmatrix} \lambda_{10} \\ \lambda_{20} \\ \lambda_{30} \end{pmatrix} = \begin{pmatrix} 0.7 \\ 0.65 \\ 0.6 \end{pmatrix},$$
$$\begin{pmatrix} v_{11e} \\ v_{21e} \\ v_{31e} \end{pmatrix} = \begin{pmatrix} 0.85 \\ 0.8 \\ 0.75 \end{pmatrix},\ \begin{pmatrix} v_{12e} \\ v_{22e} \\ v_{32e} \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0.75 \\ 0.7 \end{pmatrix}.$$

Country $1'$, $2's$ and $3's$ populations are respectively 100, 200, and 300. The depreciation rates of physical capital are the same in country 1 and country 2, and are different between country 3 and the other two countries. The total factor productivities are different between the economies. The total factor productivity of country 1's (2's) any sector is higher than country 2's ($3's$) corresponding sector. Country 1's man (woman) utilizes human capital more effectively than country 2's man (woman). Country 2's man (woman) utilizes human capital more effectively than country 2's man (woman). Country 1 (2) has higher propensity to save than country 2 (3). Woman in any country has a higher propensity to stay at home than man of the same country. Woman in any country has a lower propensity to receive education than man of the same country. The depreciation rates of human capital vary between the countries. All the groups exhibit the same return to scale effect in human capital accumulation. We specify the initial conditions as follows

$$z_1(0) = 0.05,\ \bar{k}_2(0) = 8.4,\ \bar{k}_3(0) = 3.6,\ H_{11}(0) = 4.4,\ H_{12}(0) = 3,\ H_{21}(0) = 2.9,$$
$$H_{22}(0) = 2.1,\ H_{31}(0) = 1.3,\ H_{32}(0) = 0.9.$$

The motion of the system is given in Figure 1. In the figure the national incomes and the global income are defined as follows

$$Y_j \equiv F_{ji} + p_{js} F_{js} + p_{jm} F_{jm} + p_{je} F_{je}, \quad Y \equiv Y_1 + Y_2 + Y_3 .$$

The global income and wealth slightly fall in association with rising rate of interest and falling wage rates. Country $1's$ national income rises and the other two countries' national incomes fall over time. The national labor force of and capital stocks employed by country 1 rise. The national labor forces of and capital stocks employed by the other two countries fall. There are also economic structural changes within and between countries as illustrated in Figure 1. The time distributions vary over time. It should be noted that the specified conditions $\alpha_{jq} = \alpha_{ji}$ implies that the prices are constant and not affected by the preferences.

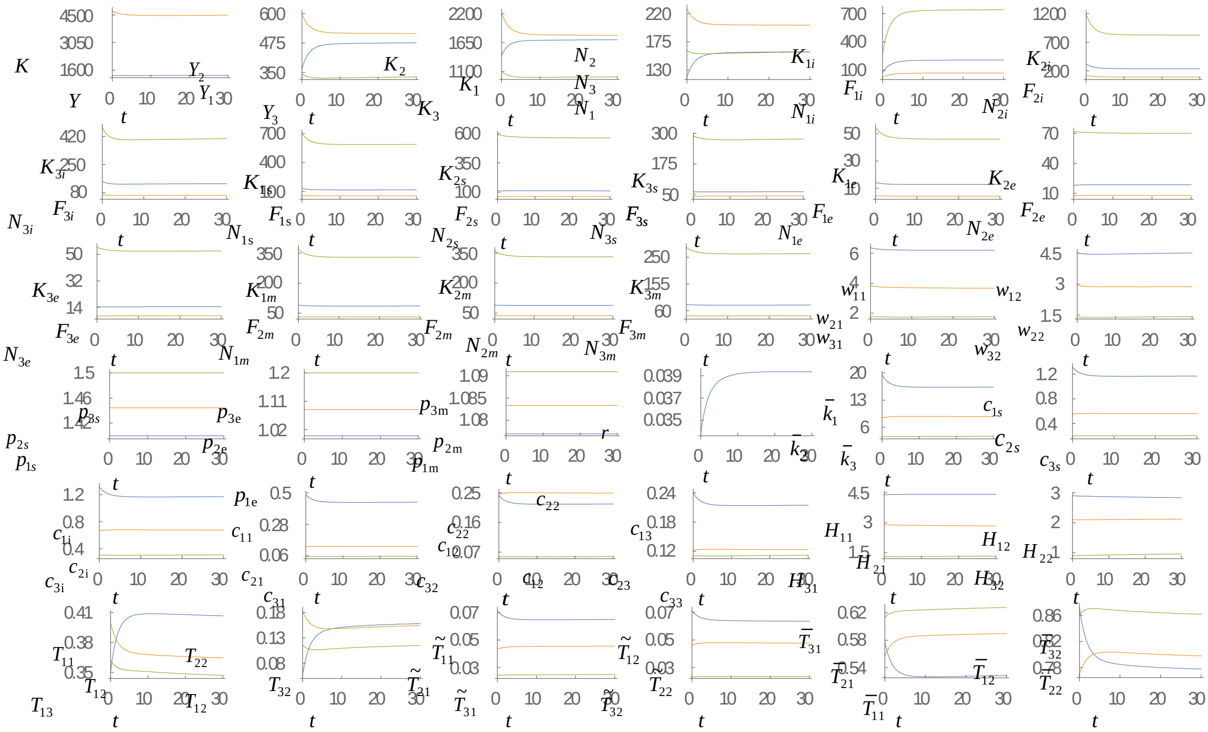

Figure 1. The Motion of the Global Economy

From Figure 1 we observe that the system becomes stationary in the long term. Following the procedure in the lemma, we calculate the equilibrium values of the variables as follows

$$Y = 1332.7, \quad K = 4532.4, \quad r = 0.039,$$

$$\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 480 \\ 513 \\ 340 \end{pmatrix}, \quad \begin{pmatrix} K_1 \\ K_2 \\ K_3 \end{pmatrix} = \begin{pmatrix} 1722 \\ 1783 \\ 1027 \end{pmatrix}, \quad \begin{pmatrix} N_1 \\ N_2 \\ N_3 \end{pmatrix} = \begin{pmatrix} 160.4 \\ 200.4 \\ 162.9 \end{pmatrix}, \quad \begin{pmatrix} \tilde{K}_1 \\ \tilde{K}_2 \\ \tilde{K}_3 \end{pmatrix} = \begin{pmatrix} 1655.6 \\ 1742.6 \\ 1134.2 \end{pmatrix}, \quad \begin{pmatrix} F_{1i} \\ F_{2i} \\ F_{3i} \end{pmatrix} = \begin{pmatrix} 208.6 \\ 235.9 \\ 139.3 \end{pmatrix},$$

$$\begin{pmatrix} F_{1s} \\ F_{2s} \\ F_{3s} \end{pmatrix} = \begin{pmatrix} 118.3 \\ 111.4 \\ 63 \end{pmatrix}, \quad \begin{pmatrix} F_{1m} \\ F_{2m} \\ F_{3m} \end{pmatrix} = \begin{pmatrix} 86.4 \\ 88.8 \\ 80.9 \end{pmatrix}, \quad \begin{pmatrix} F_{1e} \\ F_{2e} \\ F_{3e} \end{pmatrix} = \begin{pmatrix} 12.9 \\ 18.6 \\ 14.7 \end{pmatrix}, \quad \begin{pmatrix} N_{1i} \\ N_{2i} \\ N_{3i} \end{pmatrix} = \begin{pmatrix} 69.7 \\ 92.1 \\ 66.8 \end{pmatrix}, \quad \begin{pmatrix} N_{1s} \\ N_{2s} \\ N_{3s} \end{pmatrix} = \begin{pmatrix} 55.3 \\ 62.8 \\ 45.3 \end{pmatrix},$$

$$\begin{pmatrix} N_{1m} \\ N_{2m} \\ N_{3m} \end{pmatrix} = \begin{pmatrix} 31.1 \\ 37.6 \\ 42.3 \end{pmatrix}, \quad \begin{pmatrix} N_{1e} \\ N_{2e} \\ N_{3e} \end{pmatrix} = \begin{pmatrix} 4.31 \\ 7.87 \\ 8.43 \end{pmatrix}, \quad \begin{pmatrix} K_{1i} \\ K_{2i} \\ K_{3i} \end{pmatrix} = \begin{pmatrix} 748.2 \\ 819.7 \\ 421.3 \end{pmatrix}, \quad \begin{pmatrix} K_{1s} \\ K_{2s} \\ K_{3s} \end{pmatrix} = \begin{pmatrix} 594 \\ 559 \\ 286 \end{pmatrix}, \quad \begin{pmatrix} K_{1m} \\ K_{2m} \\ K_{3m} \end{pmatrix} = \begin{pmatrix} 333.7 \\ 334 \\ 267 \end{pmatrix},$$

$$\begin{pmatrix} K_{1e} \\ K_{2e} \\ K_{3e} \end{pmatrix} = \begin{pmatrix} 46.2 \\ 70 \\ 53.2 \end{pmatrix}, \begin{pmatrix} w_{11} \\ w_{21} \\ w_{31} \end{pmatrix} = \begin{pmatrix} 6.24 \\ 3.62 \\ 1.8 \end{pmatrix}, \begin{pmatrix} w_{12} \\ w_{22} \\ w_{32} \end{pmatrix} = \begin{pmatrix} 4.58 \\ 2.88 \\ 1.47 \end{pmatrix}, \begin{pmatrix} p_{1s} \\ p_{2s} \\ p_{3s} \end{pmatrix} = \begin{pmatrix} 1.4 \\ 1.44 \\ 1.5 \end{pmatrix}, \begin{pmatrix} p_{1m} \\ p_{2m} \\ p_{3m} \end{pmatrix} = \begin{pmatrix} 1.08 \\ 1.08 \\ 1.09 \end{pmatrix},$$

$$\begin{pmatrix} p_{1e} \\ p_{2e} \\ p_{3e} \end{pmatrix} = \begin{pmatrix} 1 \\ 1.08 \\ 1.2 \end{pmatrix}, \begin{pmatrix} H_{11} \\ H_{21} \\ H_{31} \end{pmatrix} = \begin{pmatrix} 4.45 \\ 2.79 \\ 1.38 \end{pmatrix}, \begin{pmatrix} H_{12} \\ H_{22} \\ H_{32} \end{pmatrix} = \begin{pmatrix} 3.19 \\ 2.12 \\ 1.01 \end{pmatrix}, \begin{pmatrix} c_{1i} \\ c_{2i} \\ c_{3i} \end{pmatrix} = \begin{pmatrix} 1.18 \\ 0.67 \\ 0.32 \end{pmatrix}, \begin{pmatrix} c_{1s} \\ c_{2s} \\ c_{3s} \end{pmatrix} = \begin{pmatrix} 1.18 \\ 0.56 \\ 0.21 \end{pmatrix},$$

$$\begin{pmatrix} c_{11} \\ c_{21} \\ c_{31} \end{pmatrix} = \begin{pmatrix} 0.44 \\ 0.13 \\ 0.06 \end{pmatrix}, \begin{pmatrix} c_{12} \\ c_{22} \\ c_{32} \end{pmatrix} = \begin{pmatrix} 0.22 \\ 0.25 \\ 0.06 \end{pmatrix}, \begin{pmatrix} c_{13} \\ c_{23} \\ c_{33} \end{pmatrix} = \begin{pmatrix} 0.22 \\ 0.12 \\ 0.12 \end{pmatrix}, \begin{pmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \bar{k}_3 \end{pmatrix} = \begin{pmatrix} 16.6 \\ 8.71 \\ 3.78 \end{pmatrix}, \begin{pmatrix} T_{11} \\ T_{21} \\ T_{31} \end{pmatrix} = \begin{pmatrix} 0.401 \\ 0.362 \\ 0.344 \end{pmatrix},$$

$$\begin{pmatrix} T_{12} \\ T_{22} \\ T_{32} \end{pmatrix} = \begin{pmatrix} 0.16 \\ 0.16 \\ 0.12 \end{pmatrix}, \begin{pmatrix} \tilde{T}_{11} \\ \tilde{T}_{21} \\ \tilde{T}_{31} \end{pmatrix} = \begin{pmatrix} 0.065 \\ 0.046 \\ 0.025 \end{pmatrix}, \begin{pmatrix} \tilde{T}_{12} \\ \tilde{T}_{22} \\ \tilde{T}_{32} \end{pmatrix} = \begin{pmatrix} 0.063 \\ 0.047 \\ 0.024 \end{pmatrix}, \begin{pmatrix} \bar{T}_{11} \\ \bar{T}_{21} \\ \bar{T}_{31} \end{pmatrix} = \begin{pmatrix} 0.53 \\ 0.59 \\ 0.63 \end{pmatrix}, \begin{pmatrix} \bar{T}_{12} \\ \bar{T}_{22} \\ \bar{T}_{32} \end{pmatrix} = \begin{pmatrix} 0.77 \\ 0.79 \\ 0.86 \end{pmatrix}.$$

It is straightforward to calculate the nine eigenvalues as follows

$$-0.6,\ -0.45,\ -0.18,\ -0.035,\ -0.035,\ -0.034,\ -0.021,\ -0.021,\ -0.019\ .$$

This implies that the world economy is stable. This implies that we can effectively conduct comparative dynamic analysis.

4 Comparative Dynamic Analysis

We simulated the motion of the dynamic system. This section carries out comparative dynamic analysis. As we can follow the motion of the global economy, it is straightforward to provide transitory and long-term effects of changes in any parameter on the global economy. We now introduce a variable $\bar{\Delta}x(t)$ to stand for the change rate of the variable $x(t)$ in percentage due to changes in the parameter value.

A rise in the total factor productivity of country 1's industrial sector

First we now study effects of an improvement of productivity in country $1's$ industrial sector. We allow the total factor productivity to be changed as follows $A_{1i}: 1.4 \Rightarrow 1.41$. The results are plotted in Figure 2. The global capital falls initially and rises in the long term. The global income rises. Country 1 employs more capital. The other two countries employ less capital initially and more in the long term. The improvement in country $1's$ industrial sector results in expansion of the sector in the country. More output is produced and more labor and capital inputs are employed. The output levels and labor and capital inputs of the other two countries' industrial sectors are reduced over time. Country 1 produces less services and employs less input factors initially and country 1 produces more services and employs more input factors in the long term. Country 1's education sector employs less labor force. It produces less and employs less physical capital initially and produces more and employs more capital in the long term. The prices of services, education and country 1's global commodity are increased and the prices in the other two countries are almost not affected. Country 1's wage rates are increased and the other two countries' wage rates are slightly affected. Country 1's human capital levels are slightly reduced. In the long term the rate of interest and the time distributions are slightly affected.

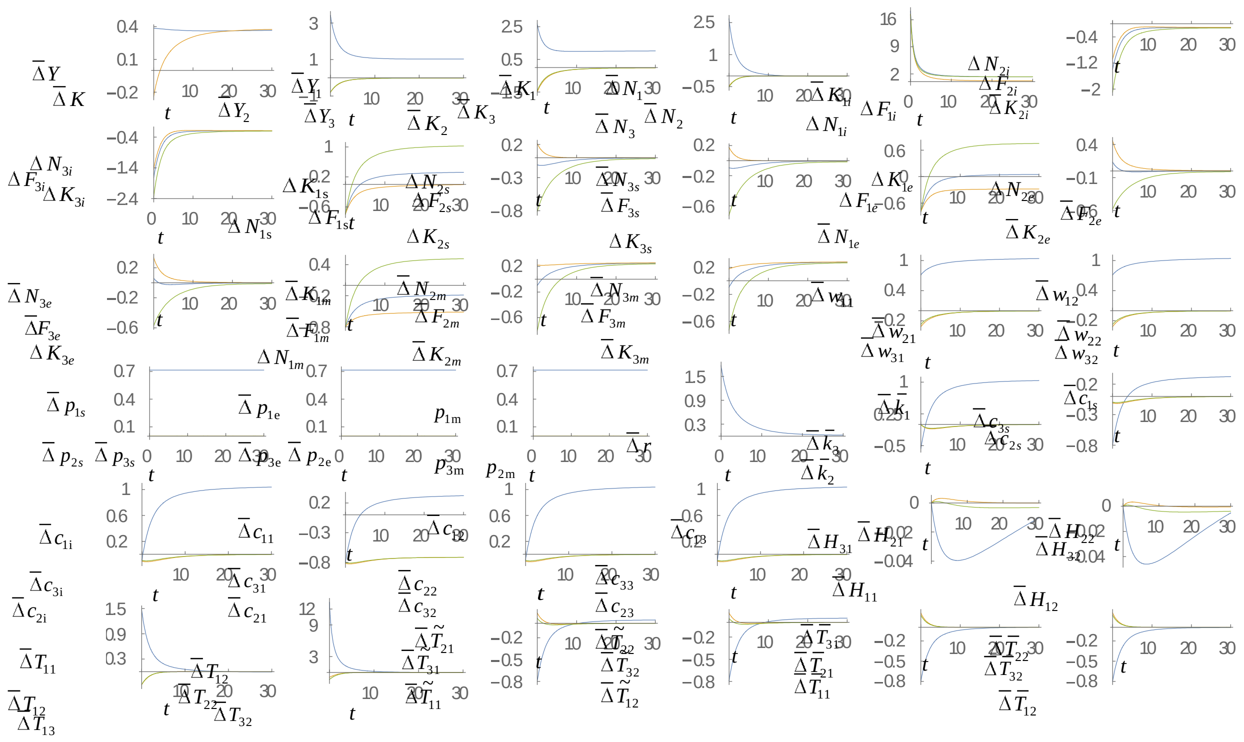

Figure 2. A Rise in the Total Factor Productivity of Country 1's Industrial Sector

We also increased the productivity of country 1's service sector as follows: $A_{1s}: 1 \Rightarrow 1.1$. It is demonstrated that the change has no impact on the system, except causing a proportional rise in the price of country 1's service.

The productivity of country 1's education sector being enhanced

We now allow the productivity in country $1's$ education sector to be increased: $A_{1e}: 1.4 \Rightarrow 1.5$. The results are plotted in Figure 3. The global capital and income fall initially and rise in the long term. The global income rises. Both woman and man's human capital are enhanced in country 1. The levels of human capital are slightly affected in the other two countries. The rate of interest falls and the wage rates in all the economies are increased. The price of country 1's education sector is reduced and all the other prices are slightly affected. Country 1 employs less capital initially and more capital in long term. The other two countries employ more capital. Country 1 has lower income initially and higher income in long term. The other two countries have higher incomes. Both the man and the woman in country 1 spend more time on education and reduce labor hours. Man and woman in the other two economies reduce leisure time. Economic structural changes occur in all the three economies as illustrated in Figure 3.

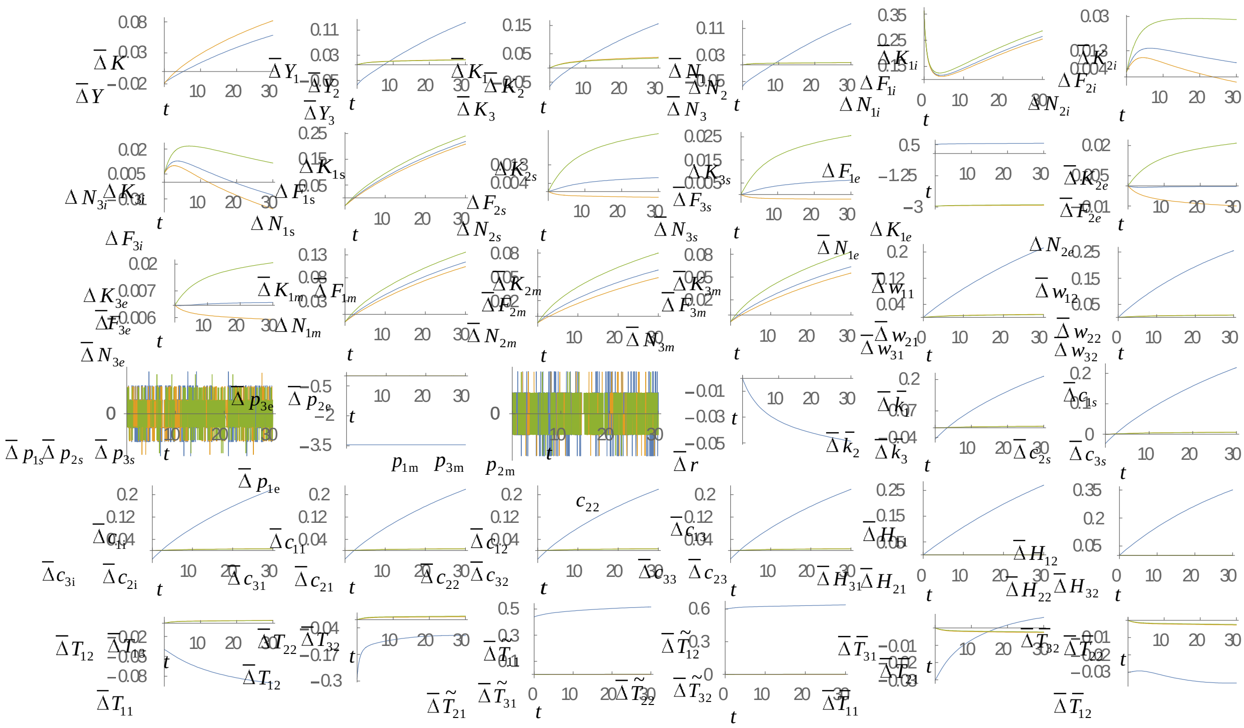

Figure 3. The Productivity of Country 1's Education Sector Being Enhanced

Country 2 increasing its propensity to consume country 1's global commodity

We now study effects of the following change in country 2's propensity to consume country 1's global commodity: $\gamma_{201} : 0.01 \Rightarrow 0.02$. The results are plotted in Figure 4. Country 2's households consumes more country 1's global commodity. The other two countries' households also consume more country 1's global commodity. The output and two input factors of country 1's global commodity are increased. The rate of interest is increased and the wage rates in all countries are reduced. The global income rises. The global capital rises initially and falls in the long term. Country 2's income rises and the other two countries' incomes fall. Country 2's human capital levels of man and woman are increased. Country 1's human capital levels of man and woman are increased. Country 2's human capital levels of man and woman are increased. Country 3's human capital levels of man and woman are slightly affected. The woman and man in country 1 work long hours and work less hours in the other two countries. Country 2 has more labor force and the other two countries have less labor force.

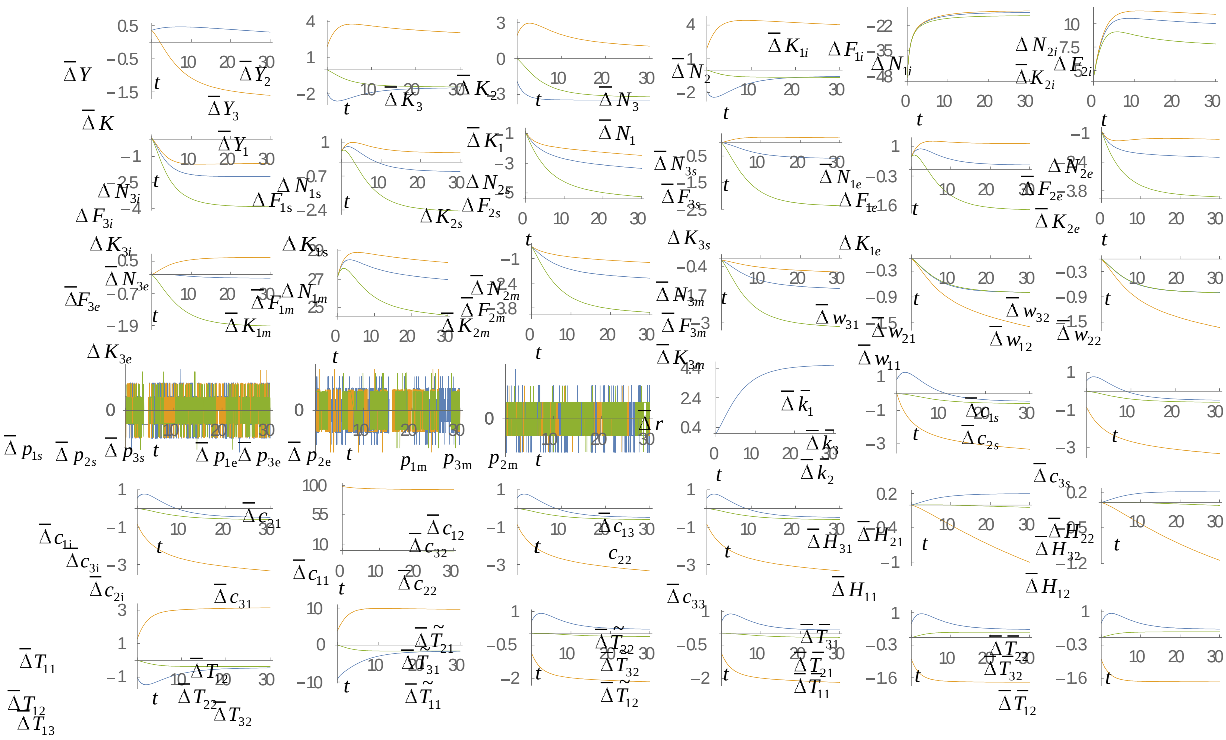

Figure 4. Country 2 Increasing Its Propensity to Consume Country 1's Global Commodity

In country 1 man's propensity to stay at home being enhanced

There is an immense body of empirical and theoretical literature on economic growth with time distribution between home and non-home economic and leisure activities. Although our study does not explicitly model endogenous changes in preferences, we can examine effects of changes in preferences on the economic system. We now examine the following change in the man's propensity to stay at home, $\sigma_{101}: 0.14 \Rightarrow 0.15$.

As the man in country 1 prefers to staying at home longer, his work time and education time are reduced and his leisure time is increased. The woman in country 1 works longer hours and reduces her leisure and education time. The global wealth and capital are reduced. Country 1's national income falls and the other two countries' national incomes are slightly affected. The prices are not affected. The economic structural changes are illustrated in Figure 5.

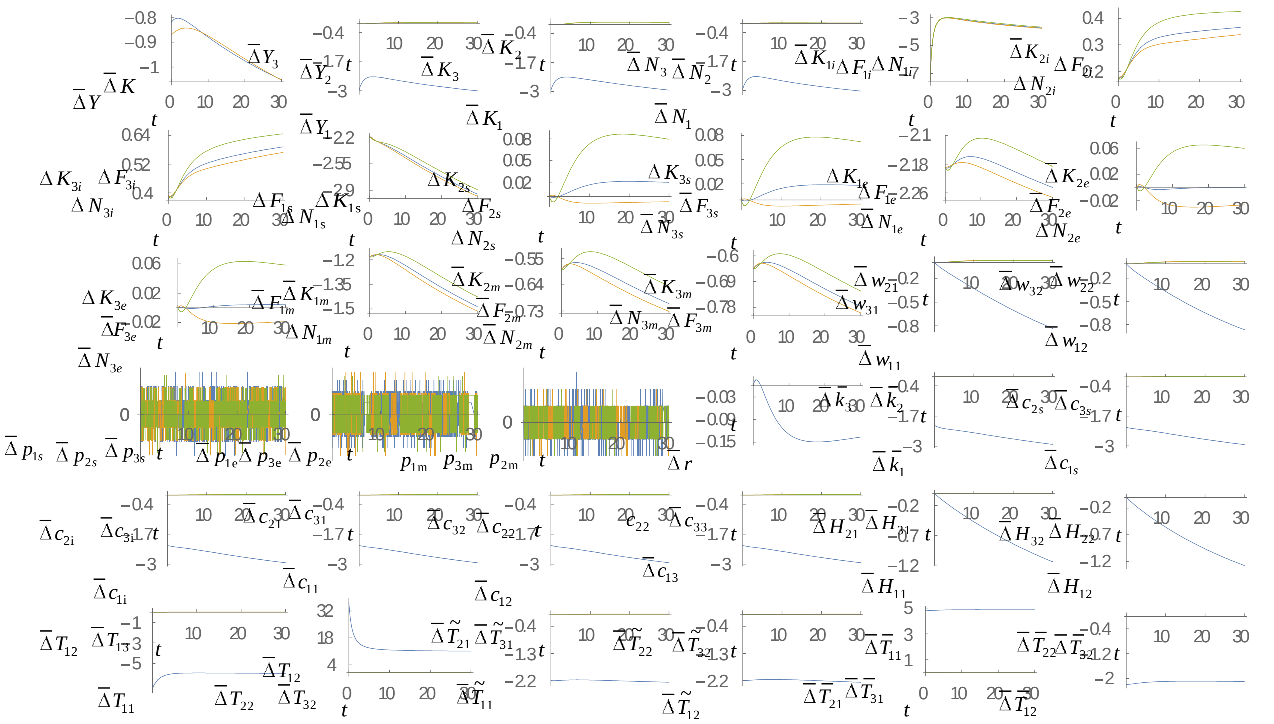

Figure 5. In Country 1 Man's Propensity to Stay at Home Being Enhanced

In country 1 the woman's propensity to receive education being enhanced

Stotsky (2006: 18) observes that "the neoclassical approach examines the simultaneous interaction of economic development and the reduction of gender inequalities. It sees the process of economic development leading to the reduction of these inequalities and also inequalities hindering economic development." A main character of modern economies is an increasing proportion of female population gets higher education. We now try to provide some insights into this issue by allowing the woman's propensity to receive education in country 1 to increase in the following way: $\eta_{102} : 0.015 \Rightarrow 0.018$. The simulation results are demonstrated in Figure 6. As country 1's woman is more interested in receiving education, her time for education is increased and for leisure is reduced. Her human capital is increased. It is interesting to notice that she spends less time on work initially and more time in the long term. Country 1's man accumulates more human capital. His leisure time and education time are reduced initially and enhanced in the long term. His work time is increased initially and reduced in the long term. The wage rates in country 1 are increased. The wage rates in the other two economies are slightly reduced. The global economy is improved in the long term. Country 1's income rises and the other two countries' incomes fall slightly. In the long term country 1's household consumes more goods and has more wealth. The micro variables of the other two countries are slightly affected. The macroeconomic variables of the national economies are strongly affected by the preference change, as illustrated in Figure 6.

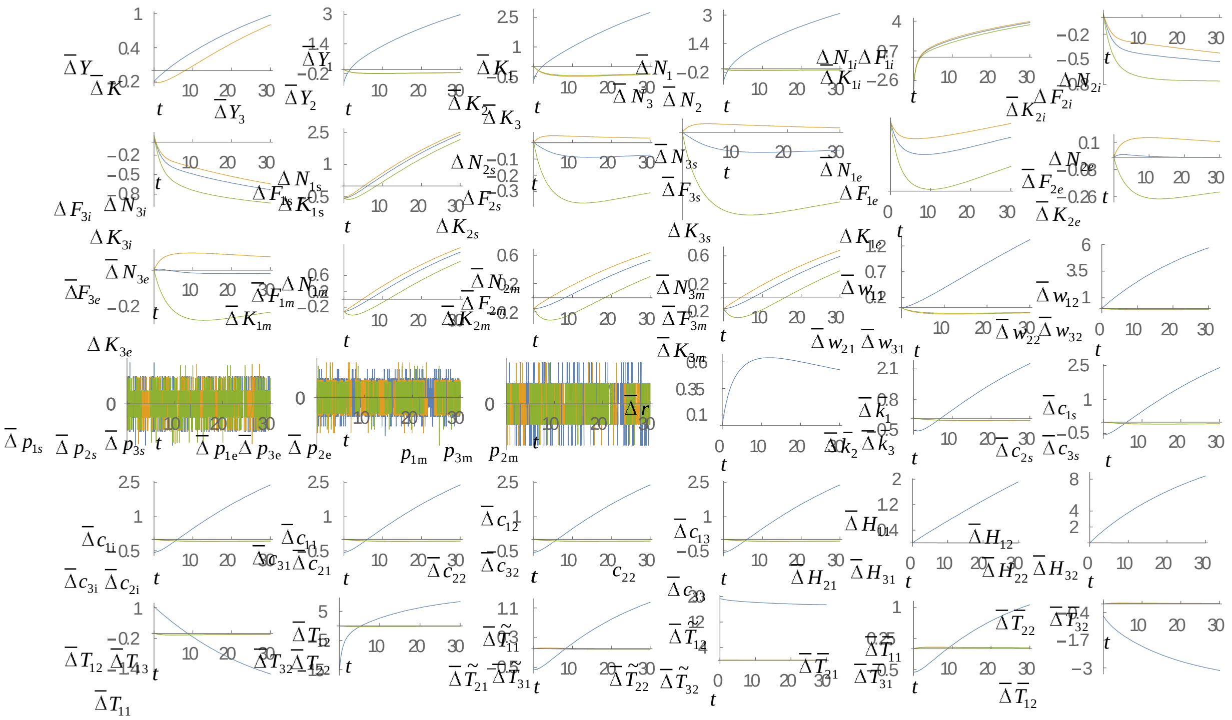

Figure 6. In Country 1 Woman's Propensity to Receive Education Being Enhanced

Country 1's population being enhanced

We now allow country 1's population to be increased as follows $\bar{N}_1 : 100 \Rightarrow 105$. The simulation results are plotted in Figure 7. The rise in country $1's$ population increases the global total income and wealth. Country 1's income and capital employed are increased, and country 2's and 3's incomes and capital employed are slightly affected. The time distributions are not affected in the long term. The wage rates are reduced and rate of interest is increased. The prices are not affected. In country 1 man's and woman's human capital levels are reduced. The consumption levels per household are slightly affected in the long term. Although their microeconomic variables are slightly affected by the population change in country 1, the macroeconomic variables of countries 2 and 3 are strongly affected as illustrated in Figure 7.

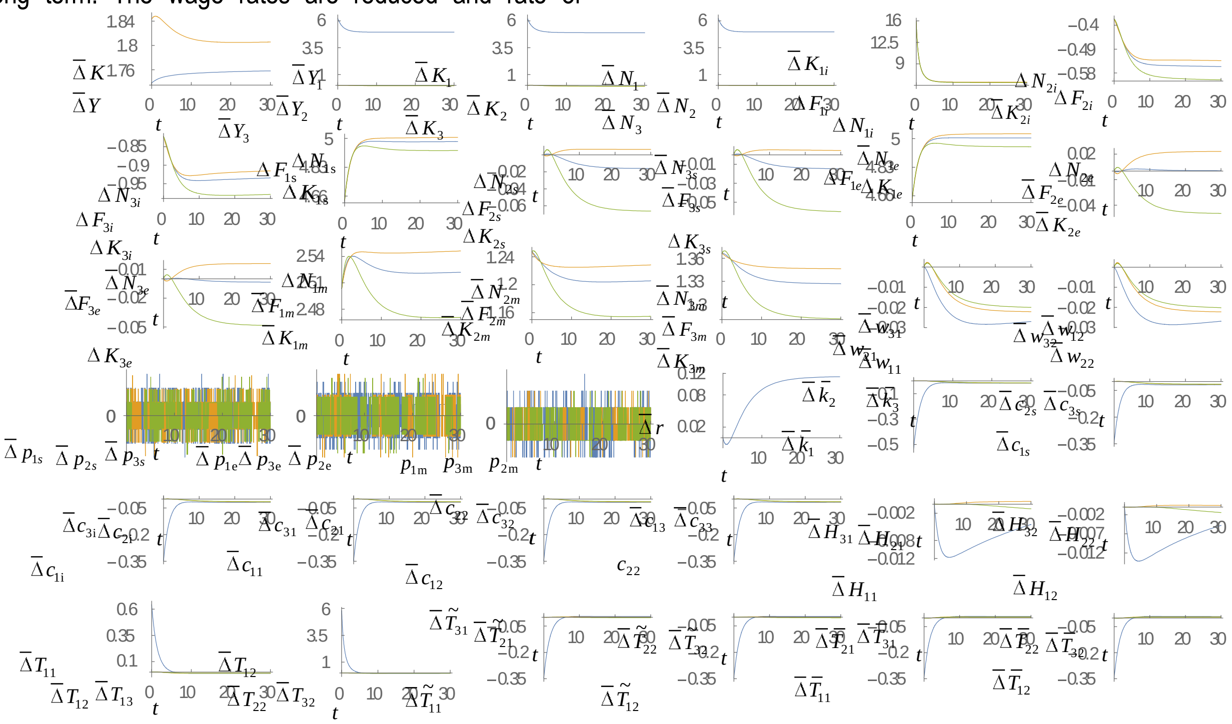

Figure 7. Country 1's Population Being Enhanced

5. Concluding Remarks

This paper studied the role of preferences and technological differences between any number of national economies in determining the dynamics of capital stocks, gender-differentiated human capital, pattern of trade and gender-differentiated and time distribution in a reformed H-O model of international trade. The paper built the trade model with endogenous wealth and human capital accumulation and labor and capital distribution between sectors and between countries under perfectly competitive markets and free trade. The model is an integration of the H-O model, the Solow-Uzawa neoclassical growth model, the Uzawa-Lucas two-sector model, and the Oniki-Uzawa trade model. The model synthesized these well-known economic models with Zhang's utility function to determine household behavior. We emphasized the impact of the gender-differentiated preferences and human capital utilization efficiencies upon the gender-differentiated time distribution and human capital and wage rates. We took account of learning by education in modeling human capital accumulation. We simulated the model for the economy to demonstrate existence of equilibrium points and motion of the dynamic system. We also examined effects of changes in the total factor productivity of a country's industrial sector, the productivity of country's education sector, one country's propensity to consume another country's global commodity, man's propensity to stay at home, woman's propensity to receive education, and a country 1's population. We might generalize and extend the model in different directions. For instance, we might get more insights from further simulation. Our comparative dynamic analysis is limited to a few cases. The Solow model, the Uzawa two-sector growth, the Uzawa-Lucas two-sector model, and the Oniki-Uzawa trade model are most well-known models in the literature of growth theory. Many limitations of our model become apparent in the light of the sophistication of the literature based on these models. We may generalize and extend our model on the basis of the traditional literature in the neoclassical growth model and trade. We may introduce tariffs into the model. This study does not consider public goods and services. There are many trade models which explicitly emphasize technological change and innovation as sources of global growth.

Appendix: Proving the Lemma

We now derive dynamic equations for global economic growth. From equations (3), we have

$$z_j \equiv \frac{r+\delta_{jk}}{w_j} = \frac{\bar{\alpha}_{ji} N_{ji}}{K_{ji}} = \frac{\bar{\alpha}_{jm} N_{jm}}{K_{jm}} = \frac{\bar{\alpha}_{js} N_{js}}{K_{js}} = \frac{\bar{\alpha}_{je} N_{je}}{K_{je}}, \tag{A1}$$

where $\bar{\alpha}_{jq} \equiv \alpha_{jq} / \beta_{jq}$. From (3) and (A1), we have

$$r(z_j) = \frac{\alpha_{ji} A_{ji} z_j^{\beta_{ji}}}{\bar{\alpha}_{ji}^{\beta_{ji}}} - \delta_{jk}, \quad w_j(z_j) = \frac{r+\delta_{jk}}{z_j}. \tag{A2}$$

From (A2) we have

$$z_j = \left(\frac{r(z_1)+\delta_{jk}}{\alpha_{ji} A_{ji}}\right)^{1/\beta_{2i}} \bar{\alpha}_{ji}. \tag{A3}$$

From (3) and (A1)

$$p_{jq} = \frac{w_j z_j^{\alpha_{jq}}}{\beta_{jq} A_{jq} \bar{\alpha}_{jq}^{\alpha_{jq}}}, \quad q = m, s, e. \tag{A4}$$

From (4) and the definitions of $\bar{p}_{jg}$ and $\bar{y}_j$, we have

$$w_{jg} = H_{jq}^{\theta_{jq}} w_j, \quad \bar{p}_{jg} = p_{je} + w_{jg}, \quad \bar{y}_j = (1+r)\bar{k}_j + \hat{w}_j, \tag{A5}$$

where $\hat{w}_j \equiv w_{j1} T_0 + w_{j2} T_0$. From (10) and (A1) we have

$$\bar{\alpha}_{ji} N_{ji} + \bar{\alpha}_{jm} N_{jm} + \bar{\alpha}_{js} N_{js} + \bar{\alpha}_{je} N_{je} = z_j K_j . \quad \text{(A6)}$$

From (7) and (12) we have

$$N_{js} = \bar{n}_{js} \bar{k}_j + \tilde{n}_{js} , \quad \text{(A7)}$$

where we use (3) and (A5) and

$$\bar{n}_{js} \equiv \frac{(1+r)\chi_j \beta_{js} \bar{N}_j}{w_j}, \;\; \tilde{n}_{js} \equiv \frac{\hat{w}_j \chi_j \beta_{js} \bar{N}_j}{w_j}.$$

From (3) and (13), we have

$$\tilde{T}_{j1} + \tilde{T}_{j2} = \frac{w_j N_{je}}{\beta_{jq} \bar{N}_j p_{je}}.$$

Insert (7) in the above equation

$$N_{je} = \bar{n}_{je} \bar{k}_j + \tilde{n}_{je} , \quad \text{(A8)}$$

where we use (A5) and

$$\bar{n}_{je} \equiv (1+r)\left(\frac{\eta_{j1}}{\bar{p}_{j1}} + \frac{\eta_{j2}}{\bar{p}_{j2}}\right)\frac{\beta_{je} \bar{N}_j p_{je}}{w_j}, \;\; \tilde{n}_{je} \equiv \left(\frac{\eta_{j1}}{\bar{p}_{j1}} + \frac{\eta_{j2}}{\bar{p}_{j2}}\right)\frac{\beta_{je} \hat{w}_j \bar{N}_j p_{je}}{w_j}.$$

Insert (A7) and (A8) in (12)

$$N_{ji} = N_j - \tilde{n}_j - \bar{n}_j \bar{k}_j - N_{jm} , \quad \text{(A9)}$$

where

$$\bar{n}_j \equiv \bar{n}_{js} + \bar{n}_{je} , \;\; \tilde{n}_j \equiv \tilde{n}_{js} + \tilde{n}_{je} .$$

From (1) and $T_{jg} + \tilde{T}_{jg} + \bar{T}_{jg} = T_0$, we have

$$\frac{N_j}{\bar{N}_j} = H_{j1}^{\theta_{j1}} T_0 + H_{j2}^{\theta_{j2}} T_0 - H_{j1}^{\theta_{j1}} \bar{T}_{j1} - H_{j2}^{\theta_{j2}} \bar{T}_{j2} - H_{j1}^{\theta_{j1}} \tilde{T}_{j1} - H_{j2}^{\theta_{j2}} \tilde{T}_{j2} , \quad \text{(A10)}$$

Insert (A10) in (6)

$$N_j = \bar{n}_{0j} - \hat{n}_{0j} \bar{y}_j , \quad \text{(A11)}$$

where

$$\bar{n}_{0j} \equiv \left(H_{j1}^{\theta_{j1}} + H_{j2}^{\theta_{j2}}\right) \bar{N}_j T_0, \quad \hat{n}_{0j} \equiv \left(\frac{\sigma_{j1}}{w_j} + \frac{\sigma_{j2}}{w_j} + H_{j1}^{\theta_{j1}} \frac{\eta_{j1}}{\bar{p}_{j1}} + H_{j2}^{\theta_{j2}} \frac{\eta_{j2}}{\bar{p}_{j2}}\right) \bar{N}_j .$$

Insert (A5) in (A11)

$$N_j = \bar{n}_{0j} - \hat{n}_{0j} \hat{w}_j - (1 + r)\hat{n}_{0j} \bar{k}_j . \qquad \text{(A12)}$$

Insert (8) in (13)

$$\sum_{j=1}^{J} \gamma_{jq} \bar{y}_j \bar{N}_j = p_{qm} F_{qm} . \qquad \text{(A13)}$$

Insert (3) in (A13)

$$N_{jm} = \gamma_{0j} + \bar{\gamma}_{0j} \sum_{v=1}^{J} \gamma_{vj} \bar{N}_v \bar{k}_v , \qquad \text{(A14)}$$

where we use (A5) and

$$\bar{\gamma}_{0j} \equiv \frac{(1 + r)\beta_{jm}}{w_j}, \quad \gamma_{0j} \equiv \frac{\beta_{jm}}{w_j} \sum_{v=1}^{J} \hat{w}_v \gamma_{vj} \bar{N}_v .$$

Insert (A12) and (A14) in (A9)

$$N_{ji} = v_j - \bar{v}_j \bar{k}_j - \bar{\gamma}_{0j} \sum_{v,\, v \neq j}^{J} \gamma_{vj} \bar{N}_v \bar{k}_v , \qquad \text{(A15)}$$

where

$$v_j \equiv \bar{n}_{0j} - \hat{n}_{0j} \hat{w}_j - \tilde{n}_j - \gamma_{0j}, \quad \bar{v}_j \equiv (1 + r)\hat{n}_{0j} + \bar{n}_j + \bar{\gamma}_{0j} \gamma_{jj} \bar{N}_j .$$

Substituting (A15), (A14), (A7) and (A8) into (A6)

$$K_j = u_j + \sum_{v=1}^{J} u_{vj} \bar{k}_v , \qquad \text{(A16)}$$

where

$$u_j \equiv \frac{\bar{\alpha}_{ji} v_j + \bar{\alpha}_{jm} \gamma_{0j} + \bar{\alpha}_{js} \tilde{n}_{js} + \bar{\alpha}_{je} \tilde{n}_{je}}{z_j}, \quad u_{jj} \equiv \frac{- \bar{\alpha}_{ji} \bar{v}_j + \bar{\alpha}_{jm} \bar{\gamma}_{0j} \gamma_{jj} \bar{N}_j + \bar{\alpha}_{js} \bar{n}_{js} + \bar{\alpha}_{je} \bar{n}_{je}}{z_j},$$

$$u_{vj} \equiv \frac{(\bar{\alpha}_{jm} - \bar{\alpha}_{ji})\bar{\gamma}_{0j} \gamma_{vj} \bar{N}_v}{z_j}, \quad v \neq j .$$

From (16) we have

$$K = \sum_{j=1}^{J} K_j = u + \sum_{j=1}^{J} \tilde{u}_j \bar{k}_j , \tag{A17}$$

where

$$u \equiv \sum_{j=1}^{J} u_j , \;\; \tilde{u}_j \equiv \sum_{v=1}^{J} u_{jv} .$$

From (16) and (A17) we have

$$\sum_{j=1}^{J} \bar{k}_j \bar{N}_j = u + \sum_{j=1}^{J} \tilde{u}_j \bar{k}_j . \tag{A18}$$

Solve (A18) with $\bar{k}_1$ as the variable

$$\bar{k}_1 \equiv \varphi\left(z_1 , \{\bar{k}_j\} , \{H_j\}\right) = \left(u + \sum_{j=2}^{J} \left(\tilde{u}_j - \bar{N}_j\right) \bar{k}_j \right) \frac{1}{\bar{N}_1 - \tilde{u}_1} , \tag{A19}$$

where $\{\bar{k}_j\} \equiv \left(\bar{k}_2 , ..., \bar{k}_J\right)$. It is straightforward to confirm that all the variables can be expressed as functions of $\{H_j\}$ and $\{\bar{k}_j\}$ by the following procedure: r by (A2) $\rightarrow w_j$ by (A2) $\rightarrow z_j$ by (A3) $\rightarrow p_{jq}$ by (A4) $\rightarrow w_{jg}$ by (A5) $\rightarrow \bar{k}_1$ by (A19) $\rightarrow \bar{y}_j$ by (A5) $\rightarrow K$ by (A17) $\rightarrow K_j$ by (A16) $\rightarrow N_{ji}$ by (A15) $\rightarrow N_{jm}$ by (A14) $\rightarrow N_j$ by (A12) $\rightarrow N_{je}$ by (A8) $\rightarrow N_{js}$ by (A7) $\rightarrow K_{jq}$ by (A1) $\rightarrow F_{jq}$ by (2) $\rightarrow c_{ji}$, c_{jm}, $\bar{T}_{jg}$, $\tilde{T}_{jg}$ and s_j by (7). From this procedure and (10), we have

$$\dot{\bar{k}}_1 = \tilde{\Lambda}_0\left(z_1 , \{\bar{k}_j\} , \{H_j\}\right) \equiv s_1 - \varphi , \tag{A20}$$

$$\dot{\bar{k}}_j = \tilde{\Lambda}_j\left(z_1 , \{\bar{k}_j\} , \{H_j\}\right) \equiv s_j - \bar{k}_j , \;\; j = 2, ..., J ,$$

$$\dot{H}_{jg} = \Lambda_{jg}\left(z_1 , \{\bar{k}_j\} , \{H_j\}\right) \equiv \frac{\upsilon_{jge} \left(F_e / 2\bar{N}_j\right)^{a_{jge}} \left(H_{jg}^{\theta_{jg}} \tilde{T}_{jg}\right)^{b_{jge}}}{H_{jg}^{\pi_{jge}}} - \delta_{jgh} H_{jg} . \tag{A21}$$

Here, we don't provide explicit expressions of the functions as they are tedious. Taking derivatives of equation (A19) with respect to t yields

$$\dot{\bar{k}}_1 = \frac{\partial \phi}{\partial z_1} \dot{z}_1 + \sum_{j=2}^{J} \tilde{\Lambda}_j \frac{\partial \phi}{\partial \bar{k}_j} + \sum_{g=1}^{2} \sum_{j=1}^{J} \Lambda_{jg} \frac{\partial \phi}{\partial H_{jg}} , \tag{A22}$$

where we use (A21). Equal (A20) and (A22)

$$\dot{z}_1 = \left(\tilde{\Lambda}_0 - \sum_{j=2}^{J} \tilde{\Lambda}_j \frac{\partial \phi}{\partial \bar{k}_j} - \sum_{g=1}^{2} \sum_{j=1}^{J} \Lambda_{jg} \frac{\partial \phi}{\partial H_{jg}} \right) \left(\frac{\partial \phi}{\partial z_1} \right)^{-1} . \tag{A23}$$

In summary, we proved the lemma.

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